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What are the Perspectives for an Applied Austrian Economy?

Production Period and Cycles: Several Interpretations in a Neo-Austrian Economic Perspective

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Abstract. The aim of this contribution is to study the notion of the period of production by taking into consideration the time-consuming nature of capital. Long delays between investment expenditures and receipts of profits from capital are indeed a remarkable property of the Austrian theory of capital. The study of the great essays of the neo-Austrian capital theory modeling allows us to postulate that there are at least two large contributing groups depending upon whether the production period is endogenous or exogenous. Our work consists in showing the well-founded methodology of the first current by suggesting a neo-Austrian inerpretation of the non-steady state behavior of the standard macroeconomic model. We show that the origin of economic cycles is the potential conflict between the producer's plan of investment through the period of production and the inter-temporal choices of the consumers.

Key Words: period of production, cycles, economic growth

JEL classification: D9, C0, N0.

1. Introduction

Since the nineteen-sixties there have been various attempts to revive and unify the Austrian capital theory into a general economic model. However classifying these various contributions seems quite difficult, each author being motivated by only one or few concepts of the Austrian capital theory.

The reason for this is that the Austrian theory is quite rich. For instance, while some authors concentrated on the concept of superiority of roundabout methods, others studied the concept of impatience. Whatever the model and the methodological approach chosen, synthesizing the Austrian capital theory into a unique model is a painstaking task.

Then there appears a second difficulty about the methodological question of the use of mathematics in the Austrian theory. It is common to consider the refusal to mathematically formalize the economic theory stands out as a specific feature of the Austrian School. However widespread this point of view is not general, for example, Bhöm-Bawerk writes in his "*Positive Theory of Capital*" that "*an economist-mathematician could quite simply enclose all things appertaining to the question within one simple mathematical formula*" (1929:338). The author restricts himself to one or two simple numerical examples, but the potential for modelling in mathematical terms goes well beyond such sketchy outlines. This did not escape the attention of economists such as K. Wicksell.

Lastly, beyond the question of the synthesizing of the Austrian theory we aim to show in this contribution that the very definition of only one concept is quite complex. We will show this through the analysis of the period of production by taking into consideration the time-consuming nature of capital. Long delays between investment expenditures and receipts of profits from capital is indeed a remarkable property of the Austrian theory of capital.

Before presenting our interpretation of the production period, it may be useful to present a review of the literature about the production period, particularly, the diversity of models of the production period. This first part clearly establishes a dividing line between a positive point of view recognizing the originality of the Austrian theory and a skeptical point of view which is highly reductive. The first interpretation was introduced at the end of the nineteen fifties, the authors highlighting the originality of the Austrian theory of capital by emphasizing the notion of production period (Dorfman et al. 1958, Dorfman 1959b, Hirschleifer 1967).

Authors belonging to the second and later current of thought see no originality in the Austrian theory; Burmeister (1974, 1980) and Cass (1973). Indeed, they state that the Austrian theory is nothing more than a special case of v. Neumann's standard model (1938) or of the neo-classical theory of growth.[1] We may remark that Burmeister (1974) in his article on the neo-Austrian theory does not mention the articles of Dorfman (1958, 1959b), Hirscheifer (1967) and Neuburger (1960). Since Dorfman wrote within the framework of the first current while at the same time taking part in the conception of the famous DOSSO model (Dorfman, Samuelson, and Solow 1958) which was directly inspired by the contribution of v. Neumann (1938, 1946) this omission is quite surprising.

In addition, Solow (1961) interpreted the Austrian capital theory from the point of view of the first current. Such "theoretical dichotomy" is more than a mere historical curiosity. The authors of the first current did put forward elements enabling the dynamic of economy to be modeled differently via the notion of the production period. Our work consists in showing the well-founded nature of the first current by suggesting an Austrian interpretation of the non steady state behavior of the standard macroeconomic model. We show that the origin of economic cycles is the potential conflict between the producer's plan of investment through the period of production and the inter-temporal choices of the consumers.

In particular, we will deal with the difficulties arising from the problem of expectations of agents on an inter-temporal level. The introduction of *time* especially via the period of production prevents agents from coordinating their decisions on savings and consumption. The conjunction of the Austrian concept of production period with a non substitution between the capital and labor with the agent's inter-temporal preferences being the fundamentals of the economic cycles. The time interval brought about by the production period gives rise to the occurrence of cycles and the Austrian theory of capital does provide an original point of view in this respect.

The paper is organizes as follows: Section 2 outlines the mathematical models of the neo-Austrian capital theory; Section 3 characterizes the neo-Austrian capital theory with an endogenous production period.

2. The Mathematical Models of the Neo-Austrian Capital Theory: Methods and Hypotheses Leading to Different Properties

The debates largely concern the integration of time and especially the concept of production period. Two directions are visible: an initial current defining a pseudo production function in which time is presented as a very special quasi-factor of production. Dorfman (1958), Hirschleifer (1967) and Cass (1973) accept this simplification which enables them to account for this notion of production period. However, other authors such as Burmeister (1974), v. Weizsäcker (1971) are far more critical.

> The definition of capital as factor of production does not work in the present model (*the Böhm-Bawerk's model*) and the definition of 'time' as a factor of production as developed in the present model cannot easily be applied to the standard production model. Thus the law of higher productivity of higher roundaboutness fails to hold in the standard production function model, v. Weizsacker (1971:39).

For the later, the notion of average production period is more relevant than that of the pseudo-production function. The production period is endogenous when time is a quasi-production factor. It is indeed determined by the producer who maximizes profit. The central theme of Bhöm-Bawerk is decidedly closer to this conception. He writes "*by choosing wisely or by lengthening the detours of production over time*, *a technical increase in output can generally be arrived at*"[2] Positive Theory of Capital (1929:279). He clarifies his central theme while reminding us that he means "*wisely chosen lengthening*" (ibid.:280) by the producers.

However, the critique of the pseudo-production function means admitting that the production period is exogenous.

2.1. Exogenous Production Period Models

We will present those works which view the production period as a piece of data. We will deal first of all with the constructive work of v. Weizsäcker (1974) and Hicks (1970). Although they have this point in common, it does not prevent them from being divided on the contribution of the Austrian theory. The former presents the theory as a game of hypotheses doing little more than propping up traditional theory whereas Hicks sees a real alternative to neo-classical logic in the Austrian theory.

Secondly, we will present the very skeptical interpretation of Burmeister (1974) who sees no originality whatsoever in the Austrian theory in the sense that he sees it as nothing more than a special case of the v. Neumann theory.

2.1.1. The Production Period Completes the Study of the Stationary Condition (Weizsäcker). The v. Weizsäcker's arguments can be summarized in two points. The first deals with the analysis of the stationary condition of the economy (a regime in which the economy identically reproduces itself whatever the date in time) using the notion of average production period. The second point shows how the production period sheds new light upon the problem of the reswitching of techniques.

Comparative Static and Average Production Period in the Study of the Steady State. The contribution of v. Weizsäcker (1971) presents the average production period using an index making Bhöm-Bawerk's concept more operative. Each production is defined in an unequivocal way in relation to time. The sum of all this production will give an average production over time. The author then deals with the effects of variation in consumption or in the interest rate upon the length of production and the other variables relative to the stationary condition.

The v. Weizsäcker approach is a way to demonstrate a paradoxical situation in which the interest rate increases following a rise in savings. The increase in savings will indeed bring about an increase in the production of intermediary goods.

> The 'paradox' of capital theory—the possibility of cases when the capital stock in constant prices rises across steady states with a rising interest rate—rests crucially on the fact that the means of production are themselves produced v. Weizsäcker (1976:211).

Production Period and the Reswitching of Techniques. Apart from the contribution regarding the properties of the steady-state-equilibrium of a Neo-Austrian model, the author generalizes the notion of period of production by the definition of the average production period. By supposing that capital is not fixed, the structure of time is modeled by the distribution of the labor input versus time. The average production period corresponds to the order moment 'one' of the distribution function while order moment 'two' indicates the variation in labor distribution (cf. Appendix A).

This rather complex presentation appears as a non convincing argument to justify the neoclassical point of view on the problem of the reswitching of techniques. At each technology change, the one with a long production period will be replaced by a technology having a shorter production period when the interest rate increases. This relation would appear as in Figure 1. This shows the factor-price frontier ϕ, a function relating wages rates (w) to interest rates (r).

The interest rate $r \in r_0 + \varepsilon$, with $\varepsilon > 0$ in Figure 1 is a point of technological change as we obtain the following conditions: $w_1 = w_2 \Leftrightarrow \phi_1(r_0) = \phi_2(r_0)$ where ϕ_i denotes the

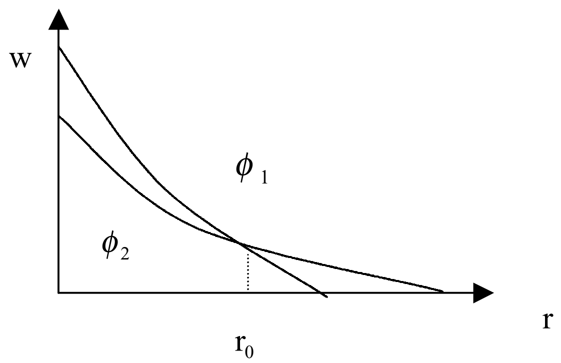


Figure 1. The factor-price frontier.

price frontier of technology i, $w_1 > w_2 \Leftrightarrow \phi_1(r) > \phi_2(r)$ for $r \in r_0 - \varepsilon$ and $w_1 < w_2 \Leftrightarrow \phi_1(r) < \phi_2(r)$ for $r \in r_0 + \varepsilon$.

The choice of technology is made depending on the interest rate and the production period. The economy will change technology at interest rate $r \in r_0 + \varepsilon$ from technology 1 to technology 2 when the interest rate increases.

The production period is a good indicator of the capital intensity of a production technique. Any substitution of one technique for another gives rise to a reduction in capital intensity when the interest rate increases. Despite the existence of counter-examples, the author claims that the argument of Bhöm-Bawerk supports the major neo-classical position. However, he does admit the implacable and original nature of the concept of production period and suggests a more operative definition which removes numerous problems referred by Fisher (1907). These conclusions do not match the interpretation made by Hicks (1970), who tries to show that the neo-Austrian tradition should lead to a theory in its own right.

2.1.2. The Period of Production as a Basis for an Alternative Theory of the Dynamics of the Economy (J.R. Hicks). The neo-Austrian theory of growth is an alternative to the Walrasian model. The sequential structure of time in the Walras model is seen by Hicks (1970)[3] to be insufficient. The definition of temporal sequences in general equilibrium makes modelling very complicated, as the author admits. The Austrian theory allows us to explain situations outside equilibrium to be accounted for. This point is particularly emphasized in the definition of technology.

2.1.2.1. A Dynamic, Alternative Model: Temporal and Technical Sequences. In the Walrasian model, technology is characterized by a matrix of coefficients (input-output matrix) defined on one and the same horizon. Technology does not therefore directly depend on time. However, the neo-Austrian perspective is different because the coefficients depend upon time. At each date, the author defines an operation as production (output) $b(t)$, as an outcome of the flow of work (input) $a(t)$.

The economic constraints are standard ($a(t)$, $b(t) \in R_+^*$) and the output of scale are supposed constant. Time t is measured starting from the date the operation began (calendar dates[4]). The operation corresponds to a plan of investment (construction and operational phases) which defined the unit of reference. The length of an operation lasts from time 0 to time U. It is active as long as its life-span does not go beyond U. The notion of operation represents not only the construction of a factory or of a machine, but also their productive use until the date the product goes out of production. The replacement of an investment marks the start of a new operation. The operations' growth rate is continuous. This rate noted as $x(T)$ at date T is not constant. These hypotheses imply the following properties:

(i) any operation whose length goes beyond U is finished. Those operations still active at date T are those having begun between T and $T - U$.

(ii) the total labor force employed at date T for all active operations is written:

$$A(T) = \int_0^U x(T - t)a(t)\,dt$$

(iii) total production at date T for all active operations is written:

$$B(T) = \int_0^U x(T - t)b(t)\,dt$$

It is interesting to note that the sum of $A(T)$ and $B(T)$ is independent from the actualization factor. If fixed wage rate w is expressed in real terms, we obtain: $b(t) - wa(t) = q(t)$.

The surplus from the operation is noted as $q(t)$. This is negative at first in the construction phase and then becomes positive only to decline until the end of the operation (cf. Appendix B). The total surplus of the economy is written thus: $B(t) - wA(t) = Q(t)$.

The characteristic of the economy is that it has a capital market with a constant interest rate ρ. The values of capital k at date θ is equal to the value of the actualized residual surplus (the surplus of θ to U) at date θ.

$$k(\theta) = \int_\theta^U q(t)e^{-\rho(t-\theta)}\,dt$$

At the outset of the operation the value of the capital is written thus:

$$k(0) = \int_0^U q(t)e^{-\rho t}\,dt$$

The capital value is nil $k(0) = 0$ in competitive equilibrium (cf. Appendix B). When k is positive, it is advantageous to begin an operation and then to immediately sell the surplus. When capital is negative, no operation is undertaken. This property of equilibrium is also defined by Hicks (1970) as *Fisher condition.* "*This condition*, $k(0) = 0$, *is substantially equivalent to Fisher's* "*rate of interest = rate of return over cost*"; *so I shall call it* ***Fisher condition***." (1970:270).

The interest rate must be equal to the output rate net of production costs in equilibrium. This condition may be written as:

$$\int_0^\theta q(t)e^{-\rho(t-\theta)}\,dt + \int_\theta^U q(t)e^{-\rho(t-\theta)}\,dt = k(0)e^{\rho\theta} = 0,$$

or as:

$$\int_\theta^U q(t)e^{-\rho(t-\theta)}\,dt = -\int_0^\theta q(t)e^{-\rho(t-\theta)}\,dt.$$

The value of the capital at date θ is equal to the sum of the value of the deficits accumulated before θ when prices are supposed constant. Fisher's condition shows the irreversible nature of the investment. The only way to enter an operation in full swing is to acquire capital. This point demonstrates that date U may be calculated according to the technical or economic life-expectancy. As the latter is often shorter it is used to define U. In other words, U is the date at which the value of capital ceases to be positive. We can deduce the following property from this convention: $k(\theta) > 0, \forall\theta \in]0, U]$ as long as $k(U) = 0$.

Date U has been chosen to maximize $k(\theta)$. The maximum is reached when $k(\theta) = 0$. If the date of the end of the operation is before or after U, the capital value is positive ($k(\theta) > 0$) or negative ($k(\theta) < 0$).

$$K(T) = \int_0^U x(T-\theta)k(\theta)d\theta = \int_0^U x(T-\theta)\int_\theta^U q(t)e^{-\rho(t-\theta)}dt\, d\theta \qquad (*)$$

In the expression (*), total capital at date T is equal to the overall capital of the active operations at this date. The first term measures the overall operations in activity whilst the second represents the present value of the capital used. In our numerical example, all operations begun starting from -0.5 to date 1.5 are active. These operations have a capital with a present value for the period defined from date 1.5 to 2 (cf. Appendix B). The derivative of expression (*) enables us to calculate net investment $K'(T)$.

We subtract $K'(T)$ from to $\rho K(T)$ and we obtain a remarkable expression:

$$\rho K(T) - K'(T) = \int_0^U [\rho x(T-\theta) - x'(T-\theta)]e^{\rho\theta}\int_\theta^U q(t)e^{-\rho t}\, dt\, d\theta$$

The term under the first integral may be written:

$$\begin{aligned}
\frac{d\big(x(T-\theta)e^{\rho\theta}\big)}{d\theta} &= \rho x(T-\theta)e^{\rho\theta} - x'(T-\theta)\, e^{\rho\theta} \\
\rho K(T) - K'(T) &= \int_0^U q(t)e^{-\rho t}\big[x(T-\theta)e^{\rho\theta}\big]_0^t\, dt \\
&= \int_0^U q(t)e^{-\rho t}[x(T-t)e^{\rho t} - x(T)]\, dt \\
&= \int_0^U q(t)x(T-t)\, dt - x(T)\int_0^U q(t)e^{-\rho t}\, dt \\
&= Q(T)
\end{aligned}$$

The total surplus of the operation is nil if Fisher's condition is satisfied (cf. Appendix B). The second integral is equal to zero in equilibrium. Once the notion of operation and Fisher's condition are in place, the author studies the dynamics of the economy's transition.

2.1.2.2. The Properties of the Dynamics of Transition. On a formal level, this work is equivalent to the study of comparative static. This point of view is nevertheless more general than that of v. Weizsäcker as the economy is not always in a stationary condition.

Transition Dynamics and Price Variation. Hicks supposes that duration U is initially fixed and examines the effect of the interest rate variation (ρ) and of the wage rate (w) upon the value of capital. The first relation is the derivative of capital compared to the interest rate:

$$\frac{\partial k(0)}{\partial \rho} = -\int_0^U tq(t)e^{-\rho t}\, dt$$

Let us put $\hat{k}(\theta) = k(\theta)e^{-\rho\theta}$ as the actualized capital value at date θ. We obtain:

$$\hat{k}(\theta) = \int_{\theta}^{U} q(t)e^{-\rho t}\, dt$$

or

$$\hat{k}(\theta) = \left[-q(t)\frac{e^{-\rho t}}{t}\right]_{\theta}^{U} = q(\theta)\frac{e^{-\rho\theta}}{\theta}$$

hence,

$$\begin{aligned}\hat{k}'(\theta) &= -q(\theta)e^{-\rho\theta}\\ \frac{\partial k(0)}{\partial \rho} &= \int_{0}^{U} t\hat{k}'(t)\, dt\\ &= [t\hat{k}(t)]_{0}^{U} - \int_{0}^{U} \hat{k}(t)\, dt\\ &= -\int_{0}^{U} \hat{k}(t)\, dt.\end{aligned}$$

In conclusion, we can say that the value of capital decreases when the interest rate rises.

When we are dealing with the effect of an increase in salary on capital value, the relationship is obtained via Fisher's condition. The wage rise may intervene at the beginning or end of a period. The operation is in its construction phase.

$$k(\theta) = \int_{0}^{\theta} -[q(t)]e^{-\rho(t-\theta)}\, dt.$$

When t falls between 0 and θ, the extra deficit is $[-q(t)]$. The capital decreases if the increase takes place at the end of the period. In fact, the wage rise reduces the effect of the surplus. This property is valid whatever the interest rate ρ. In these conditions, the reduction of duration U may make the operation profitable once more. The effect on overall capital $K(T)$, depends on the proportion of new operations compared to old ones. The effects of a wage rise are the following:

(i) the duration of the operations is reduced to recover profitability
(ii) a change in the production techniques

The second point brings two types of change:

(a) a minor change in which the new techniques alters a part of the sequences
(b) a major change in which the new techniques alter all the sequences

Hicks favors major changes as they reveal an opposite relation between wages and profits. In these conditions, maintaining profitability will mean reducing the duration of the operation (the production period).

Dynamics of Transition, Changes in Technology and Employment. Hicks then considers the effects of introducing a new technology for a given wage rate. This introduction increases the interest rate ρ and a percentage of the old operations are no longer profitable $k(0) < 0$. Those operations having begun after date 0 include the new technology which is a characteristic of the whole group (a, b, U). The old operations defined as (a^*, b^*, U^*) remain active as long as their surplus in positive. Measuring the effect of technical innovation on the volume of jobs is performed by defining the following integral:

$$A^*(T) = \int_0^T x^*(T-t)a^*(t)\,dt, \quad \forall T > 0.$$

This is the volume of jobs necessary should the technical innovation not appear. This aggregate must be compared to the volume of jobs linked to a technical innovation $(A(T))$:

$$A(T) - A^*(T) = \int_0^T x(T-t)a(t)\,dt - \int_0^T x^*(T-t)a^*(t)\,dt$$

Evaluating this complex expression leads the author to resort a presentation of the simplest case:

$$\begin{aligned} k(0) &= q_1 \int_0^M e^{-\rho t}\,dt + q_2 \int_M^U e^{-\rho t}\,dt \\ &= \left(\frac{q_1}{\rho}\right)(1 - e^{-\rho M}) + \left(\frac{q_2}{\rho}\right)(e^{-\rho M} - e^{-\rho U}) = 0 \end{aligned}$$

The capital in place from date 0 to M is used from date M to U. This may be simplified by supposing that the process is very long ($\lim e^{-\rho U} \xrightarrow{U\to\infty} 0$), hence:

$$\begin{aligned} \left(\frac{q_1}{\rho}\right)(1 - e^{-\rho M}) + \left(\frac{q_2}{\rho}\right)e^{-\rho M} &= 0 \\ q_1(1 - e^{-\rho M}) &= -q_2 e^{-\rho M} \\ \frac{q_2}{-q_1} &= (e^{\rho M} - 1) \\ \frac{(1 - wa_2)}{-wa_1} &= (e^{\rho M} - 1) \end{aligned}$$

The increase in gross output of the physical capital is equal to the interest rate per period. This expression may be written:

$$\frac{q_2}{-q_1} = (e^{\rho M} - 1)$$
$$\frac{(q_2 - q_1)}{-q_1} = e^{\rho M}$$

When we assume that the period of production is not long, we can set:

$$\begin{aligned}\left(\frac{q_1}{\rho}\right)(1 - e^{-\rho M}) + \left(\frac{q_2}{\rho}\right)(e^{-\rho M} - e^{-\rho U}) &= 0\\
\left(\frac{q_2}{\rho}\right)(e^{-\rho M} - e^{-\rho U}) &= -\left(\frac{q_1}{\rho}\right)(1 - e^{-\rho M})\\
q_2\big(1 - e^{-\rho(U-M)}\big) &= -q_1(e^{\rho M} - 1)\\
\big(q_2 - q_2 e^{-\rho(U-M)}\big) &= (-q_1 e^{\rho M} + q_1)\\
(q_2 - q_1) &= e^{\rho M}(q_2 e^{-\rho U} - q_1)\\
\frac{(q_2 - q_1)}{q_1} &= e^{\rho M}\left(\left(\frac{q_2}{q_1}\right)e^{-\rho U} - 1\right)\end{aligned}$$

The relationships between the variables are difficult to analyze, (cf. Appendix C). The duration of process U quickly becomes indeterminate compared to the working date of capital M. Fisher's condition checks for an increasing or decreasing relationship between the duration of operations and the starting date the capital U and M is used (i.e. the derivative between U and M changes sign according to the level of parameters). This property makes economic interpretation difficult.[5]

The author reveals the effect on employment according to the relative weight of the labor coefficients. During the construction phase, if the technical labor coefficient is low in comparison to the old technique, replacing the old machines will prove more costly in labor during the construction phase. The capitalist will set up machines with the same amount of capital but the decrease in the labor coefficient brings about an increase in the labor force. For equal capital in monetary terms, installing new machines requires more labor as the technical labor coefficient has fallen:

> The crucial condition is that the new machines should be expensive in labor, relatively to those they displace, and that, in spite of that, they should (of course) be more profitable (Hicks 1970:273).

However, this relationship goes beyond the results obtained from the model but Hicks considers it relevant as it matches economic history. The author continues his study by varying the date parameter. This analysis leads to what Hicks has called "the Hayek effect".

The "Hayek" Effect. The most important parameter is M, the date the capital is put into use. The Austrian economists as well as authors such as Jevons and Wicksell have emphasized this time period as it is a central property of the theory of capital. Using a simple profile, Hicks evaluates the effect of lengthening the installation phase of physical capital ($dM > 0$). If capitalists do not alter their real savings, the new process can not be set in place. It is only possible to set up a new process if there are new and real savings or if the real wage is decreased. Hicks compares this property to those highlighted by Hayek in *Prices and Production* (1931). The comparison is somewhat distant however, as Hayek formulated these mechanisms in monetary terms.

In fact, Hicks believes that the mathematical models in place are still in their research stage.

2.1.2.3. Hicks' Research Program. Hicks underlines the importance of the dynamic approach in the Austrian theory. This point should be the cornerstone of any neo-Austrian construction:

> In an Austrian theory past and future must always be distinguished (Hicks 1973:203).

The 1970 contribution is limited to technological dynamics. The author admits that this first step must be supplemented by the dynamics of prices and expectations:

> We have been very 'dynamic' on the side of inputs and outputs, but on the price-side we have been very static (Hicks 1970:279).

In an article in 1973, Hicks outlines a solution by introducing a savings function to enable the study of the dynamics of price.

He frequently returns to the difficulty of modelling and insists on the technical aspects, as in his opinion, the soundness of a theory must pass through the stage of mathematical modelling. Hicks' work remains exploratory as he changes his analytical framework on several occasions. For example, he performs a study of Fisher's condition in continuous time in 1970 and in discreet time in 1973.

Although Hicks and v. Weizsäcker are in disagreement over the interpretation of certain concepts of the Austrian tradition they do recognize their originality, albeit at very different degrees. Burmeister (1974, 1980) on the other hand, considers the Austrian theory to be of no interest whatsoever in the context of neo-classical theory.

2.1.3. Burmeister's Radical Criticism. This author shows how the Austrian model is a particular case of equilibrium in the style of v. Neumann. Burmeister's pessimism filters through in a particularly clear-cut conclusion:

> In conclusion, the neo-Austrian approach to capital theory offers no significant advantages in terms of economic theory. With a proper economic interpretation of 'good in process' as different commodities, conventional approaches encompass the neo-Austrian method as one special case (Burmeister 1980:153–154).

Apart from the total reduction in propositions and concepts, he makes reservations concerning the very relevance of this particular case. His arguments may be summarized in two points:

(i) The unrealistic character of the notion of detours in production to define capital. The hypothesis of production detour supposes that at the outset the capital good must only be a product of the labor factor. Such a hypothesis puts to one side the situation where any capital good requires labor and capital.
(ii) The production process is a "black box". The workers make undefined merchandise before the maturity date. This is an important limit as taking intermediary goods into account may well complete this model.

The two arguments deal with the need for realism in modelling. Such criticism is far from being satisfactory as the model with generations of agents—the standard for macro-economics—indeed displays these failings. The definition of the initial conditions of the generations of agents model is totally unrealistic. The interest of the model of course is that it deals with the stationary conditions of the economy. When it comes to the existence of intermediary goods, the model is very often built using one single good such as corn which is both a consumer good and a capital good (Reichlin 1986).

As for the second criticism, it could just as well have been directed at the dominant current model. Hicks (1970) replies to this by supposing that the aggregate renders the capital homogeneous and that an evaluation is carried out under a hypothesis of constant prices. Burmeister (1970), in his writings, does recognize the arbitrary character of the aggregate:

> The literature contains some discussion of the question when individual engineering relations might legitimately be aggregated into a statistical abstraction (*ibid*. 1970:9).

He finally notes that a modelling of the Austrian theory must necessarily be performed using a linear technology. But, à priori, nothing imposes such a choice as the work of Hirschleifer (1967), Dorfman (1959a, 1959b)[6] and Hennings (1997) has shown.

Whoever the writer, the role of expectations built around the notion of production is conspicuously absent from the discussions even if Hicks does see an interest in it. This role is indeed central, as Böhm-Bawerk's article (1901) on the function of savings demonstrates. Other authors such as Dorfman adopt the hypothesis of an endogenous production period whilst emphasizing the need to complete the model by taking agents' expectations into account via a savings function.

3. Modelling the Neo-Austrian Economy with an Endogenous Production Period

In this section, we wish to show how the notion of expectation via the savings function gradually came to establish itself when explaining the notion of endogenous production period. The period is endogenous in the sense that its length is determined by the producer. The latter may be more or less restricted by the technical solutions open to him. In this respect, the writings of Böhm-Bawerk leave the interpretation wide open. It is Dorfman (1959a, 1959b) who formulates the endogenous production period which is a development of the Böhm/Wicksell model. Hirschleifer completes the formulation by introducing consumer preferences. This approach is a first step in the study of the relationships between agents' expectations and the production period. But the author does not develop the study in terms of expectations as he supposes the economy to be in a stationary condition.

3.1. A Supply Model with an Endogenous Production Period

Dorfman (1959a) examines Böhm's theory through the relationship between the wage rate (w), the value of capital K_0 and the interest rate (r), put forward by Wicksell. This work highlights three points:

(i) the approximate nature of the value of capital is not always negligible
(ii) Wicksell's formalization presents a sub-system which is incomplete as such
(iii) it suffers from a serious failing in the sense that the interest rate is not defined. Indeed, one of the main aims of the *Positive Theory of Capital* is to explain how the interest rate is arrived at.

These points can be developed more clearly. We can illustrate the Böhm/Wicksell production model using the metaphor of a tree. The latter is produced thanks to the fundamental factor of labor,[7] we will call L. Labor is supplied at a constant rate in relation to the plantation date and the felling date of the tree. The production process is continuous. But it is supposed that the rate of growth in production is constant over time. The age of the trees is evenly distributed. Each tree felled is replaced. On an aggregate level, the amount of work input and output (i.e. wood production) is continuous (we will suppose in this case that it is a consumer good) and the stock of capital is constant. The simplified presentation of the model with the tree as capital does not facilitate the calculation of aggregate capital. Here we have a complex of livelihoods distributed over whole of the dates of the life-span of a tree (from the date it was planted $t = 0$ until it is felled $t = \theta$.

The production function is defined for a long-term technology $f(\ )$. The first relation Böhm/Wicksell put forward expresses the fact that the annual production growth rate per worker depends positively on the production period θ. When the hypothesis is a stationary economy, the present rate of production per capita is equal to the future rate of production (current production y_0 per head is equal to future production y_f and we have $\frac{y_0}{L} = \frac{y_f}{L}$).

According to these hypotheses, we can write that:

$$\frac{y_0}{L} = \frac{y_f}{L} \tag{1}$$

Current output y_0 is stationary in the sense that the economy recurs identically for a given growth rate of the aggregate labor force L and a given period θ.

Production per capita increases at a falling rate with period θ: $f'(\theta) > 0$ and $f''(\theta) < 0$. Equation (1) asserts that production is proportional to the labor used for a given production period. This hypothesis is in line with the hypothesis of decreasing output relating to the production period for a given volume of labor. Moreover, the greater the supply of labor, the more real capital in the form of processed goods or subsistence will be needed.

We can now return to the notion of capital. Real capital is a stock of consumer goods in the process of being manufactured. Let us suppose that $\theta = 10$ years with 100 trees and that one tenth of the trees will mature after 9 or 10 years (i.e. 10 trees), one tenth after 8 or 9 years... There is no method to measure aggregate capital because one unit of consumer goods close to the final date (10 years) is different from that at the outset of the production process (1 year). The aggregates could be measured in values by opening a market for each date (a market for 1 year mature trees, one for 2 year olds etc.). Let p_1 and p_2 be the prices of the tree at dates 1 and 2, we can then define the value of the capital. But the prices between the different dates are linked via the interest rate $p_0 = \frac{p_1}{(1+r)}$.[8] This hypothesis questions the superiority of roundabout processes compared to direct methods of production (cf. Burmeister 1974). The hypothesis of a secondary market makes investments reversible so much so that the longest production processes have an output equivalent to that of the neo-classical model. This point made by Hicks (1970), is a fundamental limit to the argument of Böhm-Bawerk according to which the increase in roundabout production (and consequently the lengthening of the production period) generally equates to a rise in output.

In his discussion concerning Böhm/Wicksell, Dorfman (1959a, 1959b) analyses the productive role of capital beginning with the stock of real goods under process. Capital value is expressed in monetary terms by Böhm/Wicksell and its measurement is defined by adding the wages paid. When calculating capital, a two-year old tree is counted twice compared to a one year old tree. The former includes twice as much labor as the latter. This relation is expressed by Eq. (2):

$$\frac{K_0}{L} = \frac{1}{2} w\theta \tag{2}$$

Capital per worker is measured by the sum needed for him to survive for a wage rate w over half the production period. In other words, each worker obtains $w\theta$ units of subsistence uniformly distributed over all the stages of production from 0 to θ. Here Eq. (2) is merely an approximation of the value of capital (cf. Appendix D). Actualization over period θ needs to be taken into account, which alters the formula considerably:

$$\frac{K_0}{L} = \frac{w}{\theta} \int_0^t e^{r\tau} d\tau = \frac{w}{r\theta}(e^{rt} - 1)$$

Integration for current projects overall may be written:

$$\frac{K_0}{L} = \int_0^\theta \frac{w}{r\theta}(e^{rt} - 1) = \frac{w}{r^2\theta}(e^{rt} - 1 - r\theta)$$

The equilibrium value for the harvest becomes:

$$f(\theta) = \frac{w}{r\theta}(e^{r\theta} - 1).$$

This is the value of the production of goods and services offered at equilibrium on the markets:

$$f(\theta) = w + r\frac{K_0}{L}.$$

Under the hypothesis of constant outputs of scale, we have the following relation:

$$f(\theta) - w = \frac{w}{r\theta}(e^{rt} - 1 - r\theta) = \frac{rK_0}{L}$$

Once we have clarified the mathematical formulation we can return to the interpretation of the notion of capital put forward by Dorfman (1959a). According to him, Böhm/Wicksell take capital K_0 as a constant. The producer is a representative agent. The community overall possesses capital which it uses by selecting the production process period θ. If the capitalist producer has a capital of 1200 florins and supposing the annual wage of a worker is 1100 florins, the number of employees for a period of one year ($\theta = 1$) is 24 and 12 when $\theta = 2$.

This calculation is relevant on an individual level as competition is such that salary w and interest rate r are given. However, this property is no longer valid on a social level because w and r are dependent on the selected period of production θ. Centralized equilibrium does not necessarily correspond to decentralized equilibrium. The variable K_0 depends on the production period as is shown by the passage to the limit in (2), $K_0 \to 0$ when $\theta \to 0$. We may conclude that capital is indeed an endogenous variable.

According to Hirschleifer, the hypothesis of the representative agent enables the distinction between the private and social dimensions to be avoided:

> Assume that there is a 'representative individual' whose productive (entrepreneurial) and consumptive decisions are a microcosm of those of the entire society. Hence, there is no need to distinguish in the symbolism between private and social magnitudes (...) (Hirschleifer 1967:194–195).

By supposing that the selected production period is never very long, so as to make linear approximation possible, the economy can hence be described using the following **S** system:

$$\frac{y_0}{L} = f(\theta) \tag{1}$$

$$\frac{K_0}{L} = \frac{1}{2}w\theta \tag{2}$$

$$f'(\theta) = \frac{1}{2}rw \tag{3}$$

$$f(\theta) = w + r\frac{K_0}{L} \tag{4}$$

This is a system of four equations with four unknowns y_0, w, r and θ. Population L and K_0 are exogenous variables. The first equation characterizes the economy's technology. The second measures capital in relation to salary whilst the third is defined using profit[9] maximization compared to the production period. The fourth represents equilibrium on the goods and services market.

The economic system is determined if and only if the capital in K_0 is exogenous (cf. Appendix E). When supposing K_0 to be exogenous, the economy becomes incomplete and we then find 5 unknowns with 4 equations. To summarize, a variable needs to be determined or an equation added:

(i) The economy may be defined in equilibrium whatever the interest rate $r \in [0, +\infty[$. The interest rate will measure the amount of capital (Solow 1961). This sum is defined as a reverse function of the interest rate. Due to the under-determination of the system (i.e. the number of equations is less than the number of variables), the interest rate needs to be determined to obtain the overall values of equilibrium. This interest rate is established on the money market. It is difficult, however, to model this choice when savings depend on both the wage rate and the interest rate. Be that as it may, such a solution does not suit Böhm-Bawerk's theory which attempts to explain how the interest rate is derived.

(ii) The model may be improved by adding an extra equation. This may be the agents' saving rate. Solow (1956) put forward the hypothesis of an exogenous savings rate. By adding the equation I = S (cf. Appendix F) the system becomes complete.

In fact, the Bhöm model that Dorfman modified describes to a supply economy. This initial attempt is improved upon by Hirschleifer (1967) who introduces a savings function via agents' inter-temporal choices.

3.2. The Model Completed by Agents' Inter-Temporal Choices in a Stationary Condition

Hirschleifer accepts that capital (subsistence funds) is no longer exogenous and adds the inter-temporal choices of the consumers. This model enables capital with a K_0 value at the stationary condition to become endogenous. The consumer saves in accordance with the interest and wage rates (Appendix F). It is supposed that these variables are given at competitive equilibrium. Each producer must choose period length and takes account of both the household savings supply and technological constraints.

Extensions of Hirschleifer's Model. The malfunctioning of the market is based on the definition of the production period. Hirschleifer emphasizes that this period θ may be presented both from a social and a private angle:

> But on the social level of analysis the wage rate w and the interest rate r will both be functions of the θ adopted; there is no justification for taking the valuation of the community's capital as invariant with respect to this decision (Hirschleifer 1967:194).

In this last case, the author outlines that each firm's length of production period is characterized by a specific wage and interest rate. Moreover each firm records a specific length of production period. So it does not seem accurate to take into account the social or average production period. From this point of view, the savings must be calculated with respect to the interest rate, which is defined by the specific firm's production period.

This property may be the cause of agents' bad coordination. For example, we may take the case where two firms have different production periods. The "social" or average period is a source of coordination defect in savings decisions.

A second lengthening may be observed through the link between the notion of production period and that of generations of agents. Equilibrium à la Hirschleifer means a stationary equilibrium of the generations of agents model with production. Integrating the savings function enables expectations to be taken into account. Agents' savings is a function of the *expected* interest rate and its volume determines the capital available in the future. But the definition of equilibrium in a stationary condition prevents the study from being undertaken in dynamic terms. It is clear from the analysis that a non-stationary condition is better suited to the theory of Böhm-Bawerk (1929:260–261). Elements from the theory of expectations in Austrian logic are interesting as they bring an original interpretation of inter-temporal equilibrium. We shall present an example of the overlapping generations model for which an exogenous period of production gives rise to a defect in the coordination between agents from the moment expectations are introduced.

4. An Interpretation of the Overlapping Generations Model within the Austrian Perspective: Production Period and Cycles

The production period creates a malfunctioning in the coordination of savings of agents born in period t and the labor supply of agents born at date $t+1$ in Reichlin's model (1986). At each period agent must forecast the labor supply for the following period of the next generation. However, there is by definition no market on which agents are able to coordinate their decisions. The hypothesis of perfect expectation is not sufficient to correct these discrepancies due to the temporal structure of production in the Austrian economy. Equilibrium can be calculated using two types of agents who are representative of consumers and producers. One single commodity is used as both a consumer and investment commodity.

Representative Consumer and Producer. Two generations of agent exist at any given period: the (young) representative agent born in the present period and the (old) agent born in the previous period. It is supposed that population is constant. The young agent offers his labor l_t and receives a wage rate w_t. The choice is summarized by the maximization program: $\max_{l_t} U(c_{t+1}, l_t) = u(c_{t+1}) - v(l_t)$ under budget constraints: $c_{t+1} = w_t l_t \hat{R}_{t+1}$ where $w_t l_t = s_t$.

The variable s_t denotes the agent's savings. The income $w_t l_t$ from the first period, the whole of which has been saved, receives interest at the *expected* rate from the second period $\hat{R}_{t+1}$.

To make the study simpler, the agent consumes in the second period only. The variable c_{t+1} denotes second period consumption. The utility function is standard: $u'(c_{t+1}) > 0$, $u''(c_{t+1}) \leq 0$ and $v'(l_t) > 0$, $v''(l_t) > 0$.

The agent judges between the utility of consuming and the decrease in utility generated by labor. We shall make the following class of utility: $u(c_{t+1}) = c_{t+1}$ et $v(l_t) = l_t^2$.

Solving the consumer's program enables us, after a few operations, to obtain the following relation: $w_t l_t R_{t+1} - 2l_t^2 = 0$.

The consumer's decisions are added to by those of the representative producer. A feature of the production function is a Leontief technology which checks the hypothesis of constant outputs of scale.

$$Y_t = \min\left[\frac{K_t}{a_1}, \frac{L_t}{a_0}\right]$$

In an equilibrium of full-employment, we have: $\frac{K_t}{a_1} = \frac{L_t}{a_0}$. These relations now need completing by the equilibrium conditions of labor and goods and services market with the aim of defining the dynamics.

Market Equilibrium and Dynamic Equilibrium. Equilibrium on the labor market is written as follows: $l_t = L_t$. Equilibrium on the goods and services market highlights the notion of production period since savings are used to finance the capital of the future period: $s_t = K_{t+1}$. By advancing the following definitions: $a_0 = a_1$ and $a \equiv \frac{1}{a_1}$.

The dynamics of the economy are defined by the following system:

$$\begin{aligned} s_{t+1} &= as_t - 2L_t^2 \\ L_{t+1} &= s_t \end{aligned}$$

This is a system of difference equation generating the equilibrium trajectories of the economy. In a first step we can calculate the unique stationary state equilibrium: $L = s$ and $s = L(a - 2L)$. It can be checked that the amount of saving is positive if and only if $(a - 2L) > 0$ and $L > 0$. These values are defined by two types of parameter: the technological parameter of the function of production (a) and the agent's utility parameter. We consider the general class of utility function $U(c, l) = c^\alpha - l^\delta$ with $0 < \alpha \leq 1$, $\delta > 1$. In our example we have implemented simulations with the following numerical values $\alpha = 1$ and $\delta = 2$ for the utility function.

The proof of the existence of cycles is based on the standard mathematical theory of the Hopf bifurcation (c.f. Guckenheimer and Holmes (1983)). We compute the characteristic polynomial of the Jacobian matrix evaluated at the stationary state and we establish sufficient conditions for complex roots at a bifurcation point.

Among these conditions the economic properties are illustrated through the numerical example in Appendix H. We assume that the wage rate is relatively higher than the interest rate. Since the labor supply is rising with respect to wage rate we can conclude to an increase of the amount of labor in the production. This growth of labor will stimulate the saving. So the rate of growth will continue to increase in this way (c.f. Figure in Appendix H). This leads to an increase of the wage rate and a decrease of the interest rate. The process is valid so long as the increase of the amount of the saving balances the decrease of the interest rate. But this process is reversed when the capital income becomes too low. So the saving declines and leads to slowdown of the growth. It can be noticed that, the conjunction of the Austrian concept of period of production with a non substitution between capital and labor with the agent's intertemporal preferences are the fundamentals of the economic cycles. The time interval brought about by the production period gives rise to the occurrence of cycles.

4.1. Concluding Remarks

The relevance of the neo-Austrian theory from a modelling point of view is extremely high and reveals rather unexpected elements.

The value of the theory lies in the diversity of the models on offer. The notion of production period is at the very heart of the debate. We have shown that there are at least two large contributing groups depending upon whether the production period is endogenous or exogenous. The exogenous production period gives rise to three distinct extensions. The first interpretation notes that the theory of Böhm-Bawerk complements the standard theory of stationary conditions. The second, on the other hand, sees here a very real alternative to the dynamic of Walras. The technical difficulties are nevertheless considerable. The third interpretation reducing the whole of the Austrian theory to a single case of v. Neuman's model, seems to be of little interest.

These interpretations are all very different with a certain number of authors ignoring the attempts at modelling. Even Burmeister in his summarizing article forgets to mention the contributions of Dorfman, Hirschleifer . . .

The second set of contributions accepts the hypothesis of an endogenous production period. The neo-Austrian model thus becomes a supply model. Variables such as wages and the interest rate depend on the production period at an aggregate level. This point leads the authors to complete the model by adding an equation. Hirschleifer offers to add a savings function to account for the expectations of agents. This expression of intertemporal choices defined in a stationary condition is a sharp reminder of the model with generations of agents. Our contribution is that of interpreting the model with generations as developed for example by Reichlin, within the Austrian perspective of Böhm/Wicksell. The existence of cycles is an interesting case from an Austrian approach. It does not seem to be special case. The production period and the non substitution between capital and labor are frequently modeled by some authors such as Hicks, Bernholtz et alii. We establish that the conjunction of this time structure of production with the intertemporal agent's preferences can explain the existence of economic cycles. This approach facilitates the understanding of the model and its possible extensions. So it may be interesting to evaluate the economic properties of an economy with three periods in a future research. It appears from this that the production period generates problems of co-ordination between agents. This malfunctioning may explain complex cycles or dynamics. Writers such as MacCallum (1983) attribute these complex dynamics to the purely mathematical properties of the models. This point becomes highly debatable when an Austrian analytical grid is chosen in which the dynamic properties of the model are a good reflection of what is happening in the economy.

Appendix A

This appendix give a synthesis of the v. Weizsäcker's book (1971) chapters 6 and 7. The model is presented in the continuous time version for the sake of simplicity. The final output is produced with labor only $\{\alpha(T)\}$where T is the distance between the beginning and the end of the production. The current value of the labor at the date T the output is written: $we^{rT}\alpha(T)$where w stands for the real wage. The total cost of production is written:

$w \int_0^\infty e^{rT}\alpha(T)dT$. The price is equal to one ($p = 1$), we obtain:

$$w = \frac{1}{\int_0^\infty e^{rT}\alpha(T)dT} \tag{A}$$

The period of production is:

$$T = \frac{\int_0^\infty e^{rT}\alpha(T)T\,dT}{\int_0^\infty e^{rT}\alpha(T)dT}.$$

We differentiate (A) corresponding to r and we get the following relation:

$$0 = \frac{dw}{dr}\int_0^\infty e^{rT}\alpha(T)\,dT + w\int_0^\infty Te^{rT}\alpha(T)\,dT$$
$$\frac{dw}{dr} = -\frac{w\int_0^\infty Te^{rT}\alpha(T)\,dT}{\int_0^\infty e^{rT}\alpha(T)\,dT} = -wT$$

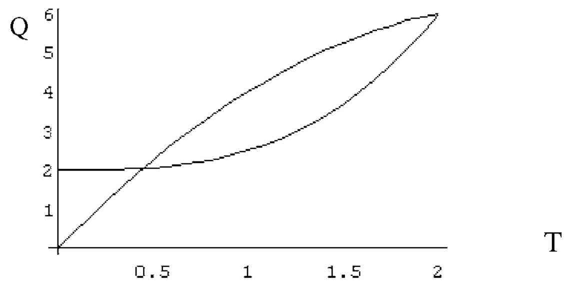
The second moment is given by the derivative of index T corresponding to t:

$$\begin{aligned}\frac{dT}{dr} &= \frac{\left(\int_0^\infty e^{rT}\alpha(T)T\,dT\right)\left(\int_0^\infty e^{rT}\alpha(T)T^2dT\right)}{\left(\int_0^\infty e^{rT}\alpha(T)dT\right)^2}\\ &\quad - \frac{\left(\int_0^\infty e^{rT}\alpha(T)T\,dT\right)\left(\int_0^\infty e^{rT}\alpha(T)T\,dT\right)}{\left(\int_0^\infty e^{rT}\alpha(T)dT\right)^2}\\ &= \frac{\int_0^\infty e^{rT}\alpha(T)T^2dT}{\int_0^\infty e^{rT}\alpha(T)\,dT} - T^2\end{aligned}$$

Appendix B: A Simulation of the Period of Production: The Hicksian Case

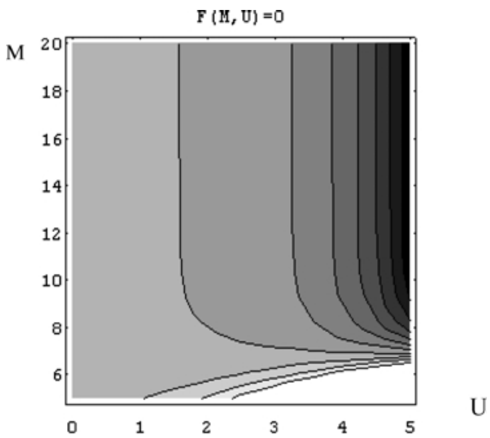
Case $r = n$, $U = 2$, $r = 1.37766$ and $w = 1$.
The capital value at $t = 0$ when $r = 1.37$: $k(0) = 0$.
The current value of capital at $t = 1$: $k(1) = 0.779921$.
The capital of an operation at the end of the period of production: $k(U) = k(2) = 0$.
The total capital at time $T = 1.5$ is: $K(T) = K(1.5) = 1.978$.
$r.K(T) = r.K(1.5) = 2.72501$,
$w.A(T) = w.A(1.5) = 12.6963$,
$B(T) = B(1.5) = 12.6963$,
$K'(1.5) = 2.72501$,
We verify the Fisher's condition:
$rK(1.5) - K'(1.5) = 0$,
$B(1.5) - wA(1.5) = 0$.

For example, the value of capital at $t = 0$ when $r = 0.9$ is different from zero, we obtain: $k(0) = 0.224827$.



Appendix C

The preceding relation is defined as an implicit function between M, U and ρ but the geometry of this system is complex. The following numerical example reveals this difficulty:



The figure is implemented with the following numerical values: $\rho = 0.989$, $q_1 = 0.011$ and $q_2 = 10$. We observe that the relation between M and U is not monotonic.

Appendix D: The Linear Approximation of the Böhm-Bawerk/Wicksell's Model

The production is defined on the time sequence $d\tau$, the wage is paid at the rate $\frac{w}{\theta}d\tau$.

The accumulated wage from 0 to t is written:

$$\frac{w}{\theta}\int_0^t d\tau = \frac{wt}{\theta}.$$

We can integrate on the length of production and we obtain:

$$\frac{K_0}{L} = \frac{w}{\theta}\int_0^\theta t d\tau = \frac{1}{2}w\theta.$$

The last expression corresponds to the Eq (2). The term of error is small when θ is not too long. From the standard linear analysis we know that $e^{r\theta} \approx 1 + r\theta$.

Appendix E

We can verify that the economy is well defined with the system **S**:

$$\frac{y_0}{L} = f(\theta) \tag{1}$$

$$\frac{K_0}{L} = \frac{1}{2}w\theta \tag{2}$$

$$f'(\theta) = \frac{1}{2}rw \tag{3}$$

$$f(\theta) = w + r\frac{K_0}{L} \tag{4}$$

The four unknowns are y_0, w, r et θ. The population L and K_0 are constant. From Eq. (2), we write the wage rate depending on the period of production: $w = w(\theta)$ the Eq. (3) is: $f'(\theta) = 1/2rw(\theta) \Rightarrow r = r(\theta)$. Using (4), we can substitute $w(\theta)$and $r(\theta)$ to obtain: $f(\theta) = w(\theta) + r(\theta)K_0/L \Rightarrow \theta^*$. Equilibrium solutions are written as follow: $y_0^*/L = f(\theta^*)$, $r^* = r(\theta^*)$, $w^* = w(\theta^*)$, θ^*.

Appendix F

The system **S** is completed with the introduction of the saving function:

$$s(w, r) = \frac{K_0}{L} \tag{5}$$

The five unknowns variables are y_0, w, r, θ et K_0. We assume that the population L is constant. From (2) and (5), we get $K_0 = K_0(w, \theta) \Rightarrow s(w, r) = \frac{K_0(w,\theta)}{L} \Rightarrow w = w(w, \theta)$. Using the expression of the wage in (3) and the interest rate can be written as a function of the period of production: $r = r(w(r, \theta), \theta) \Rightarrow r = r(\theta)$. We have the following relation: $f'(\theta) = \frac{1}{2}r(\theta)w \Rightarrow w = w(\theta)$ and $K_0(w(\theta), \theta) = K_0(\theta)$. The Eq (4) is written as:

$$f(\theta) = w(\theta) + r(\theta)\frac{K_0(\theta)}{L} \Rightarrow \theta^*.$$

The values at the equilibrium are: $\frac{y_0^*}{L} = f(\theta^*)$, $r^* = r(\theta^*)$, $w^* = w(\theta^*)$, $K_0^*(\theta^*)$, θ^*

Appendix G

The model is completed with a saving function. We can take into account agent's intertemporal preferences. The saving function $s(w, r)$ is defined through the consumer's inter-temporal program. The Hirschleifer's model is written as follow:

$$\frac{y_0}{L} = f(\theta) \tag{1}$$

$$\frac{K_0}{L} = \frac{1}{2} w\theta \tag{2}$$

$$f'(\theta) = \frac{1}{2} rw \tag{3}$$

$$f(\theta) = w + r\frac{K_0}{L} \tag{4}$$

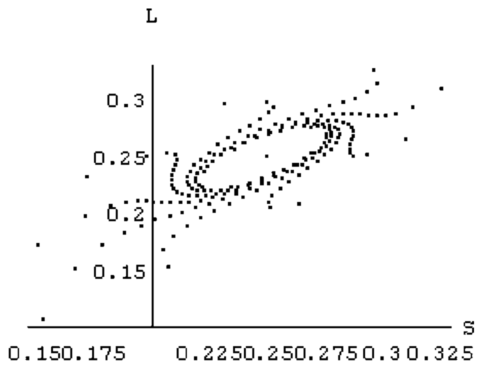
$$s(w, r) = \frac{K_0}{L} \tag{5}$$

$$\underset{c_o, c_f}{\text{Max}}\, U(c_o, c_f) \tag{6}$$

$$c_o + \frac{c_f}{(1+r)} = w \tag{7}$$

This economy is characterized by 7 equations and 7 unknowns y^*, r^*, w^*, K_0^*, θ^*, c_o, c_f.

Appendix H: Economic Cycles



Simulation is implemented ($t = 200$) with the following parameter's value $a = 1.52$ with Mathematica 2.2.1. The unique non-trivial steady state is defined by $S = 0.25$ and $L = 0.25$.

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Notes

1. Cass [1972 (1973)], one of the most eminent figures of the theory of optimal growth, shows that the fundamental properties of the neo-Austrian theory may be obtained within the framework of the most basic neo-classical hypotheses.
2. The author clarifies that this fundamental law had a purely empirical justification. He adds on several occasions that the explanation for this relationship lies in physical science indicating that "the reason for this is more of physical than of political economics" (1889b:100). This law is a simple empirical generalization which is liable to exceptions. In this sense, it is a purely positive theory of capital.
3. This point must nevertheless be treated delicately as Malinvaud (1953) deals with the sequential aspects of a temporal economy in general equilibrium. The author comes back to this point in his micro-economics teaching (1983). His contribution is also mentioned by Negishi (1985), Cass (1973) and Weizsäcker (1971).
4. If the operation begins in 1899, t is measured from this year onwards.
5. The author readily admits this point (cf. Hicks 1970, footnote:274).
6. This choice was not made by Dorfman (1959a, 1959b) who took part in the work on the famous DOSSO model, defined using a linear technology.
7. Let us not forget that for Böhm-Bawerk there are only two fundamental production factors: labor and land (the forces of nature). Capital as an indirect means of production is a derivative of these two factors. It can not thus receive payment as such. The interest rate can not be the payment of capital.
8. To simplify we suppose the interest rate to be constant. The structure of markets in an inter-temporal framework is discussed in Malinvaud (1983) chapter 10B§1.
9. The producer's program is written $\max_\theta \pi(\theta) = f(\theta) - w - rK_0/L$ under constraint $K_0/L = 1/2w\theta$. Pure profit is equal to zero in a hypothesis of perfect competition.

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